Friday #3 Wave Propagation

Wave propagation on ropes and strings depends on the two ingredients of inertia and linear restoring force. For a rope or string we define its inertia in terms of how much mass there is for a given length. We will use the symbol ***W*** for this, and it is measured in units of kilograms per meter. The restoring force depends on the tension force – how tightly it is strung. For tension force we will use the symbol ***F*** and it is measured in Newtons= $Kg \cdot m / \sec^2$.

The wave speed is given by:

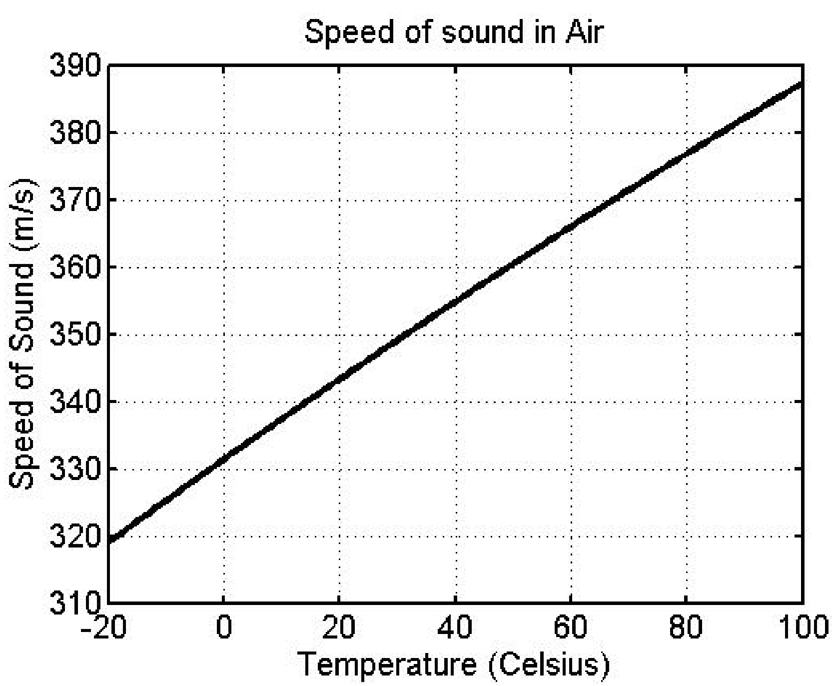
$$S = \sqrt{\frac{F}{\mathrm{W}}}$$

The ratio of ***F/W*** has the units of $\frac{Kg \cdot m \cdot m}{\sec^2 Kg} = (\frac{m}{\sec})^2$ so its square-root has the units of speed.

There is a close similarity between this formula and the formula for the speed of sound in air. The linear restoring force is provided by the air pressure and the inertia is provided by the mass density of the air. The in the ratio, the number density of atoms cancels out so that the main dependence is on the absolute air temperature *T* and the average mass per atom *M*.

So the speed of sound in air is:

$$. S = \sqrt{\frac{\gamma \kappa \mathrm{T}}{\mathrm{M}}} \approx (331.3 + 0.6 * T_{Celcius}) \frac{m}{s}$$



Friday 3
The slinky puzzle.

Consider transverse waves on a slinky. For a slinky of length $\boldsymbol{L}$ and wave speed $\boldsymbol{S}$ write down a formula for how long will it take for a wave to travel from one end to the other? This is the transit time $\boldsymbol{t}$.

$$\boldsymbol{t = \frac{L}{S}}$$

Now here is the puzzle: When the length of the slinky is doubled by stretching it out, how much will the transit time change?

Before you answer think about four things:

1) How much does the distance of travel change?

It doubles.

2) How much does the tension change?

The tension doubles because the slinky will be twice as stretched and it has a linear restoring force.

3) How much does the mass density change?

The mass of the slinky will be spread out over twice the distance, so the mass density drops in half.

4) How much does the speed change?

$S = \sqrt{\frac{F}{W}}$ **If we use F' to designate the new tension force then *F'=2F*. The new mass density *W'* is equal to *W/2*. The new speed, *S'*, is equal to:**

$$\boldsymbol{S' = \sqrt{\frac{F\prime}{W\prime}} = \sqrt{\frac{2F}{\frac{1}{2}W}} = \sqrt{\frac{4F}{W}} = 2\sqrt{\frac{F}{W}} = 2S\ldots}$$ **so it is twice as fast.**

So what is the answer**? The transit time does not change because the distance has doubled but the speed doubles as well.**

Would your answers be different if we were talking about the longitudinal wave? **No.**